Set Theory

Ling 120C: Semantics I
August 25, 2025

> "Sure, speakers of a language can learn word meanings. But how do they know what entire sentences mean without memorizing them? How does anyone ever figure out how the word meanings combine?!"
> *-someone about to discover compositionality, probably*

- **goal**: to understand how we go from individual word meanings to the meaning of a proposition
  - We need a way to isolate the variables we're studying, like in a laboratory
  - We need our model to deal with individual entities like 'Simone', content words like 'black', and function words like 'a'
  - One method for this is set theory!

1 Set-theoretic notation

1.1 The basics

> Simone and Travis are students. They are visiting a national park. Simone saw a black bear, but Travis did not.

(1) Variables

a. capital letters (A, B, C...) or curly brackets { } represent **sets** — S: the set of students; B: the set of black bear-seers

b. Ø is the **empty set**

c. lowercase letters (a, b, c...) represent **members** of sets — s: Simone; t: Travis

(2) Defining sets

a. {s, t} — the set containing (only) Simone and Travis
   $\{x \mid x \text{ is a student}\}$ — the set of students

b. $s \in B$ — Simone is a member of set B
   $t \notin B$ — Travis is not a member of set B

c. $S = \{s, t\} = \{x \mid x \text{ is a student}\}$

d. $B = \{s\} = \{x \mid x \text{ saw a black bear}\}$

Useful tips

- Sets are unordered, so if two sets have the same members, they're the same set.
- It's helpful (but not mandatory) to use the letters words start with for sets and their members.
- We aren't dealing with the multiverse, just one situation or possible way (part of) the world could be.
- A singleton set (set with one member) is not the same as the member itself.

1.2 Set relations

- Our model is pretty limited if we can only describe single sets.
- How do we express meanings like conjunction and disjunction in set theory? Introducing... **subsets**!

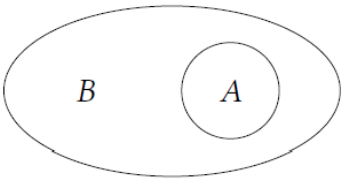
(3) Subsets

a. $B \subseteq S$ — B is a subset of S
$S \subseteq S$ — S is a subset of S
$S \nsubseteq B$ — S is not a subset of B

b. $S \subset B$ — B is a proper subset of S
$S \not\subset S$ — B is not a proper subset of S

- If B is a subset of S, every member of B is also a member of S.
  - S can have members that aren't in B... but it doesn't have to. Every set is a subset of itself.
  - If you want to specify that the two sets aren't equal, you can use a proper subset.
- Did you notice...?
  - Subset relations have truth values, just like p & q. That means they are propositions.
  - A subset **entails** the set that includes it!

(4) Set inclusion (the subset relation) Kroeger (2019): 233



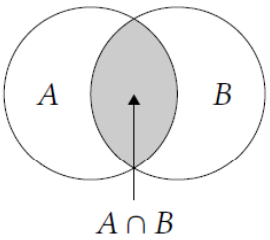
$A \subseteq B$

1.3 Set operations

- Note: The subset *relation* creates propositions. Set *operations* create new sets from combinations of other sets, so they don't have truth values.
- You already know one operation: identity (=). We'll look at three more: intersection, union, and difference.
- The **intersection** of two sets S and B is the set that includes all (and only) the elements that are members of both S and B.
  - You can think about this like a non-propositional cousin to conjunction. They are related concepts (both hats) but not identical (rounded vs. pointy).

(5) $S \cap B$ — the intersection of S and B
$S \cap B = \{s, t\} \cap \{s\} = \{s\}$

(6) Set intersection Kroeger (2019): 234

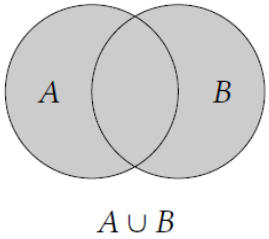


$A \cap B$

- The **union** of two sets S and B is the set that includes any elements that are members of S and/or B.
  - You can think about this like a non-propositional cousin to disjunction (rounded vs. pointy cups).
  - Helpfully, the symbol looks like the letter U.

(7) S ∪ B — the union of S and B
S ∪ B = {s, t} ∪ {s} = {s, t}

(8) Set union — Kroeger (2019): 234

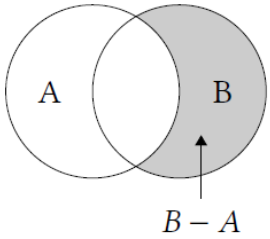


$A \cup B$

- The **difference** between two sets S and B is the set that includes all the elements that are in S but not in B.
  - How could we represent exclusive *or* in set theory?

(9) S − B — S minus B
S − B = {s, t} − {s} = {t}

(10) Set difference — Kroeger (2019): 235



$B - A$

2 Practice and application

Translate from propositional logic into set-theoretic notation.

(11) Big Bird is a bird or a man.

p: Big Bird is a bird. — propositional logic
q: Big Bird is a man.
$p \vee q$

b: — set theory
B:
M:

Now do the same thing but for exclusive *or*.